QUANTUM LIFE: INTERACTION, ENTANGLEMENT, AND SEPARATION*

Individual objects are not fixed but fluctuating, constantly responding to their surroundings, bundles of relationships, rather than settled points.[1]

When distinct quantum systems interact, they appear to lose their individuality. In almost all cases, once systems are interacting, they no longer have a proper (or "pure") quantum state at all. Moreover, the quantum state of the merged or "composite" system involves statistical correlations among the components that seem to go beyond anything obtainable from the systems that entered into the interaction, even when those systems become spatially distant from one another. Erwin Schrödinger[2] coined the term *entanglement* (*Verschränkung*) to describe this situation. Revising the old-fashioned marriage vow ("What God has joined together let no man put asunder"), entanglement seems to imply that not even God can separate quanta once merged. We ask here whether this is really so. Specifically, we ask whether entangled quantum systems are nevertheless separable, in the sense that the statistical correlations of the composite system can be obtained from statistics of the components.

We approach this question in the way that John Bell[3] and others have approached the issue of locality. We look for a clean mathematical demonstration to settle the issue one way or another. The mysteries and puzzles over quantum locality emerge from demonstrating that when the correlations of entangled systems violate the Bell inequalities, which is almost always, they cannot be explained by introducing suitable statistical states for the components. These are states that satisfy a precise condition of local action or "locality" and a condition

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[1] T. M. Luhrmann, *Persuasions of the Witch's Craft: Ritual Magic in Contemporary England* (Cambridge: Harvard, 1989), p. 239.

[2] "Die gegenwärtige Situation in der Quantenmechanik," *Die Naturwissenschaften*, XXIII (1935): 844–49, and also "Discussion of Probability Relations between Separated Systems," *Proceedings of the Cambridge Philosophical Society*, XXXI (1935): 555–63.

[3] See, for example, *Speakable and Unspeakable in Quantum Mechanics* (New York: Cambridge, 1987).

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of statistical independence called "factorizability."[4] Here we ask the comparable question for separability. Can we introduce suitable statistical states for the components that allow the correlations of the composite system to be explained on the basis of statistics of the components? If the answer is "yes" then we regard the components of an entangled system as separable; otherwise not. Surprisingly, we show that even when the Bell inequalities are violated, which is where entanglement really shows up, the components are separable.[5] In carrying out this demonstration, we actually show something more. For the states we introduce are local (although not factorizable) and hence we can conclude that entanglement is compatible with both separability and locality. Thus during marriage, as well as before, quanta may lead a healthy life of their own.

This robust conception of quantum life has been contested. Indeed violations of the Bell inequalities have prompted a literature on the metaphysics of microscopic systems that flirts with some sort of metaphysical holism regarding spatially separated, entangled systems. The rationale for this behavior comes in two parts. The first part relies on a proof, due to Jon Jarrett,[6] that the experimentally observed breakdown of the Bell inequalities entails a breakdown of the conjunction of the constraints just mentioned. Jarrett called these two constraints *locality* and *completeness*. We prefer the terminology used above of locality and *factorizability*.[7] The first part of the rationale for metaphysical holism urges that only Jarrett's locality allows for "peaceful coexistence" between quantum mechanics and special relativity. Factorizability, it is suggested, must be jettisoned.

The second part of the rationale introduces *separability*, as above, which is regarded as the opposite, in essence, of holism. The argument

[4] See, for example, N. David Mermin, "Quantum Mysteries for Anyone," this JOURNAL, LXXVIII, 7 (July 1981): 297–308; and Arthur Fine, "Antinomies of Entanglement: The Puzzling Case of the Tangled Statistics," this JOURNAL, LXXIX, 12 (December 1982): 733–47.

[5] In the sequel, we concentrate on correlation experiments whose outcomes violate the Bell inequalities. For the type of experiments considered here, satisfaction of those inequalities is sufficient for there being a local, factorizable model—see Fine, "Hidden Variables, Joint Probability, and the Bell Inequalities," *Physical Review Letters*, XLVIII (1982): 291–95. Every such model is separable. Hence, satisfaction of those inequalities implies separability.

[6] "On the Physical Significance of the Locality Conditions in the Bell Argument," *Noûs*, XVIII (1984): 569–89.

[7] Fine, "Correlations and Physical Locality," in P. Asquith and R. Giere, eds., *PSA 1980*, Volume 2 (E. Lansing, MI: Philosophy of Science Association, 1981), pp. 535–56.

here draws crucially on the lemma that separability implies factorizability. It would follow from this lemma, the Jarrett result, and the failure of the Bell inequalities that if we want to preserve locality, we must jettison separability, and embrace its opposite—metaphysical holism. Thus blame for the failure of factorizability is shifted to the failure of separability. In this way the correlations observed in entangled systems are seen as the sign of a deep (and perhaps mysterious) holism in nature.

By establishing that merged quanta may lead a life of their own, we reject here the second part of the rationale, in particular, the lemma on which it relies. Properly conceived, there is no relation of logical entailment that flows from separability to factorizability. Our discussion here complements that of Tim Maudlin,[8] who also doubts that separability implies factorizability. Maudlin conjures up an imaginative separating scheme involving tachyons and he shows how superluminal signaling may accommodate the statistics for a special experimental configuration where factorizability fails. (Without the picture of superluminal signaling, one of us suggested a similar scheme with the idea of "randomness in harmony."[9]) In the tradition of the literature on Bell's theorem, we ask here directly whether certain kinds of structures can in principle support the quantum correlations, regardless of how they may be implemented. Thus we concentrate on investigating whether the statistical relations posited by schemes like Maudlin's can be represented in mathematical models of the experimental situation, without relying on intuitions associated with tachyons, or other physical mechanisms.

In showing how to construct local and separable models of correlation experiments whose outcomes violate the Bell inequalities, we show that holism, however deep or mysterious, is not a price that we have to pay for keeping locality. But what of nonlocal theories; for example, the Bohm theory? If separability is compatible with locality, is it also compatible with nonlocality? We do not address this issue directly since our concern here has been to assess the price of locality given the failure of the Bell inequalities. If our suggestion in section IV for understanding the quantum states is correct, however, then the Bohm theory, like quantum theory itself, is separable. Hence if

[8] *Quantum Nonlocality and Relativity* (Cambridge: Blackwell, 1994), see p. 98.

[9] See Fine, "Correlations and Physical Locality," and "Antinomies of Entanglement."

the Bohm theory is actually nonlocal, then violations of the Bell inequalities turn out to be compatible with nonlocal as well as with local models, each of which respects separability.[10]

I. BACKGROUND: THE EXPERIMENTAL SETUP

In the simple EPR-Bohm type experiments we consider here (2×2 experiments), a source emits pairs of particles (call them that for simplicity) in the singlet state ψ. After emission, the particles in each pair move off in opposite directions to spatially separate wings, the A- and B-wings. Each wing contains a detector assembly with two settings (1 and 2), corresponding to the measurement of spin orientation (in the plane perpendicular to the "path" of the particles). When the particles arrive in their respective wings, the detectors register either $+1$ or -1 (corresponding to spin up or spin down in the relevant direction). In the singlet state the probability is one-half at either wing for either $+1$ or for -1 to occur, regardless of the setting there. The probability for pairs of outcomes, the joint probability, is a simple function of the angles of orientation measured in each wing.

We want to model such a correlation experiment, by introducing a space Λ of basic "states" distributed according to some fixed probability measure on the space. (The states in Λ are often referred to as "hidden variables" but we agree with Bell that this is a misleading and silly terminology; so we avoid it here.) Each state $\lambda\in\Lambda$ determines a set of joint probabilities of the form:

$$p_\lambda{}^{AB}(x,y|i,j)$$

where A is the assembly in the A-wing, B is the other wing assembly, x and y are the measurement outcomes at A and B ($+1$ or a -1) and i and j are the settings (either 1 or 2) on A and B respectively. We require that the model recover the probabilities $P^{QM}(A_i = x,\ B_j = y|\psi)$ assigned by the quantum theory in the singlet state ψ as averages over the space Λ of basic states; that is, we require that

$$<p_\lambda{}^{AB}(x,y|i,j)>_\Lambda \ = \ P^{QM}(A_i = x,\ B_j = y|\psi)$$

Since the $p_\lambda{}^{AB}(x,y|i,j)$ completely determine the probabilities of all possible outcomes of all possible measurements, it is entirely natural to regard the entire set of these functions as the state of the composite

[10] See J.T. Cushing et al., *Bohmian Mechanics and Quantum Theory: An Appraisal* (Boston: Kluwer, 1996) for interpretive essays on the Bohm theory. The essay by M. Dickson, "Is the Bohm Theory Local?" pp. 321–30, challenges the nonlocality usually associated with Bohm.

system. This way of treating the state accords nicely with the quantum mechanical rule according to which the state function of a system is completely determined by the family of probability distributions for all observables of the system.

We can then introduce what one would naturally call the marginal probabilities at λ of individual outcomes at individual measurement stations.

$$p_\lambda{}^A(x|i,j) = \sum_y p_\lambda{}^{AB}(x,y|i,j) \text{ and } p_\lambda{}^B(y|i,j) = \sum_x p_\lambda{}^{AB}(x,y|i,j)$$

These marginals constitute the states in each wing.

Locality and factorizability. We are now in a position to define the two probabilistic constraints mentioned in the introduction. *Locality* is the constraint that at each state λ the marginal probabilities at A (respectively, B) do not in any way depend on the switch settings at B (respectively, A).[11] That is:

$$p_\lambda{}^A(x|i,j) = p_\lambda{}^A(x|i,1) = p_\lambda{}^A(x|i,2)$$

and

$$p_\lambda{}^B(y|i,j) = p_\lambda{}^B(y|1,j) = p_\lambda{}^B(y|2,j)$$

Assuming locality we can simplify the notation and write

$$p_\lambda{}^A(x|i,j) = p(A_i = x|\lambda)$$
$$p_\lambda{}^B(y|i,j) = p(B_j = y|\lambda)$$

for the local states.

The second constraint is *factorizability* (Jarrett's "completeness" and Abner Shimony's "outcome independence"). It is the constraint that at each λ the joint probabilities be the arithmetic product of their respective marginals. That is:

$$p_\lambda{}^{AB}(x,y|i,j) = p_\lambda{}^A(x|i,j).p_\lambda{}^B(y|i,j)$$

For experiments of the type considered here, the Bell inequalities constrain the difference D between the sum of any three joint probabil-

[11] Abner Shimony calls this "parameter independence"—see "Search for a Worldview Which Can Accommodate Our Knowledge of Microphysics," in J. Cushing and E. McMullin, eds., *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem* (Notre Dame: University Press, 1989), pp. 25–37.

ities (that is, the ensemble averages) and the fourth. The constraint is just that $0 \leq D \leq 1$. The experimental evidence is that, for certain settings of detectors in these experiments, the measured values for the joint probabilities violate the Bell inequality. Jarrett showed that if the inequalities were violated, then at most one of the above constraints, either locality or factorizability, could hold. At least one of them has to go. In these circumstances many think that the right strategy is to keep locality and to jettison factorizability. While that conclusion has important critics,[12] for the purposes of this article we will take it at face value. Our concern is how philosophers have come to interpret what they have perceived as the failure of factorizability. Below are some typical examples.

II. SEPARABILITY

> The fact that there exist quantum states of two-body systems which cannot be factorized into products of one-body quantum states means there is *objective entanglement* of the two bodies, and hence a kind of *holism.*[13]

> In terms of the Bell-type experimental setup, for example, we are barred from representing what we would ordinarily call the "two particles" in a given "pair" as two distinct entities; although detectable in spacelike measurement events, "they" form a single object, connected in some fundamental way that defies analysis in terms of distinct, separately existing parts.[14]

> [The source of the Bell result is] a kind of ontological holism or nonseparability..., in which spatio-temporally separated but previously interacting physical systems lack separate physical states and perhaps also physical identities.[15]

If we want to get clear on the relationship between factorizability and holism, we really need to understand holism's opposite, separability.

[12] See, for example, M. Jones and R. Clifton, "Against Experimental Metaphysics," in Peter A. French et al., eds., *Midwest Studies in Philosophy,* Volume 18 (Notre Dame: University Press, 1993), pp. 295–316; Maudlin, p. 93ff.; and J. Cushing, *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony* (Chicago: University Press, 1994), pp. 57–59.

[13] Shimony, p. 27.

[14] Jarrett, "Bell's Theorem: A Guide to the Implications," in Cushing and McMullin, pp. 60–79, here p. 79.

[15] Don Howard, "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," in Cushing and McMullin, pp. 224–53, here p. 225.

Don Howard's analysis (*ibid.*)[16] is an excellent place to start. In these works, Howard argues that the experimental violations of the Bell inequalities force us to abandon the fundamental metaphysical principle of separability and to embrace, instead, a kind of holism.

In trying to formulate a precise notion of separability, Howard draws inspiration from a passage of Albert Einstein's:

> It is characteristic of these physical things that they are conceived of as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things "lie in different parts of space."[17]

Howard takes this "requirement" offered by Einstein as a fundamental metaphysical constraint on physical systems; the principle of separability.[18] He then offers a more precise formulation: the contents of any two spatiotemporally separated regions can be considered to constitute separate physical systems.[19]

Here *separate physical systems* means:

(1) Each system possesses its own *distinct* physical state.
(2) The joint state of the two systems is *wholly determined* by these separate states.

We find this articulation of the principle of separability to be very compelling.[20] It is, however, rather abstract. In particular, in order to

[16] See also his "Einstein on Locality and Separability," *Studies in History and Philosophy of Science*, XVI (1985): 171–201; and "Locality, Separability, and the Physical Implications of the Bell Experiments," in A. van der Merwe, F. Selleri, and G. Tarozzi, eds., *Bell's Theorem and the Foundations of Modern Physics* (Singapore: World Scientific, 1992), pp. 306–14.

[17] "Quantum Mechanics and Reality," *Dialectica*, II (1948): 320–24. This translation from the original German is by Howard, "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," p. 223.

[18] Maudlin (*op. cit.*, p. 97) notes that Bell makes a similar assumption when he supposes that each region of space-time has an intrinsic state.

[19] Howard, "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," p. 226.

[20] It does seem to us that there is a third condition that needs to be brought in, namely, that the states of separate parts are relatively independent of one another. (We say 'relatively' because there may be some conserved quantities shared by both systems whose values will be linked.) So minimally one would want that you cannot get *all* the information about one system part from the state of the other. Without something like this, separability would not necessarily run counter to holism (since the whole, then, might be fully reflected in both parts, in which case we would not need their 'sum'), which surely we want it to do. Most likely Howard's notion of 'distinct' partial states is intended to do this job—if worked out. None of the rest of our argument will hinge on this small quibble.

apply this principle to a quantum system, one would need to articulate both what is meant by the *state* of the system, and what counts as a state being *wholly determined* by two other states. Howard does just this in a footnote, where he specifies how his principle is to be applied to quantum systems:

> How the joint state is determined by the separate states depends upon the details of a theory's mathematical formulation. At a minimum, the idea is that no information is contained in the joint state that is not already contained in the separate states, or alternatively, that no measurement result could be predicted on the basis of the joint state that could not be predicted on the basis of the separate states.... I define a state, λ, formally, as a conditional probability measure, $p_\lambda(x|m)$, assigning probabilities to measured results, x, conditional upon the presence of measurement contexts, m. *With states thus defined, to say that the joint state is wholly determined by the separate states is to say that the joint probability measure is the product of two separate measures (ibid.*, p. 226n, our emphasis).

As already suggested, we regard Howard's treatment of states as families of probability measures to be a perfectly sensible way to think about quantum states. As above, in a Bell-type experimental setup, the entire state of the two-particle system can be thought of as being given by a set of functions onto probability measures. We like, moreover, the way Howard lays out the basic intuition of separability—that no information is contained in the joint state that is not already contained in the separate states. The last sentence of the passage, however, the one to which we have added emphasis, stands out to us as a non sequitur. Why, after all, should we equate determination with multiplication, that is, with just the product function?

Surely the joint state is *wholly determined* by the separate states just in case there is *some* function that maps the marginals on each subsystem to joints on the composite system. If it works, the product function would do (so factorizability implies separability). But to restrict our attention to this function as the only possible candidate for separating the two states strikes us as provincial. Any function, plausibly, that maps pairs of separate states onto joint states would make the whole *determinable* by the parts. That, after all, is what a function does—a binary function *determines* an output given any pair of inputs.

Howard's treatment simply equates separability with factorizability. That equation, moreover, is not confined to Howard. Nor is it even confined to discussions of the Bell inequalities. In a recent article on the relation between Leibniz's principle of the identity of indiscernibles and the Pauli exclusion principle, Michela Massimi[21] offers the following definition of separation. Given any composite system, the states of the two component systems are "ontologically separate" just in case:

(1) Each subsystem has definite (though possibly unknown) values of a complete set of compatible observables pertaining to that subsystem alone.
(2) The afore-defined ontologically separate states of the subsystems determine wholly their joint state (*ibid.*, p. 321).

So far so good. She then goes on to say:

> *Condition 2 is known as factorizability*: the joint probability of the expectation values relative to the two subsystems is the product of the probabilities bearing respectively on each subsystem alone (p. 321, our emphasis).
>
> Factorizability is uncontroversially taken as a separability condition to the extent that all non-factorizable theories involve some kind of non-separability (p. 321n).

We beg to differ. Multiplication is not the only binary function and the failure of factorizability is perfectly compatible with a complete determination of the whole from the separate parts. As we will show, there are local models of experiments in which the Bell inequalities are violated (hence nonfactorizable models) that are indeed separable.

Of course, we understand why someone might think that quantum entanglement rules out separability, which requires that the partial states (the marginals) determine the whole state (the joints). For in an EPR-Bohm experiment, the singlet state of the total system yields a uniform distribution (50:50) as the marginal on the outcome space $\{1, -1\}$ regardless of the angle being measured in either wing. So, for example, the marginals are the same in runs of the experiment with separations of zero degrees between the spin angles measured in the wings (that is, measurements in the same direction) and in runs where the difference between the measured angles is one hundred twenty degrees. But the joint probability functions are different. In the one case, they yield probability 0 for getting identical outcomes in each wing (strict anticorrelation). In the other case, that probability is 3/8. Hence no function maps the marginals of the singlet state to

[21] "Exclusion Principle and the Identity of Indiscernibles: A Response to Margenau's Argument," *British Journal for the Philosophy of Science*, LII (2001): 303–30.

the joints. (Recall that a function determines its result, and so does not map one to many.) Looking at quantum theory this way, it seems that the crucial condition for separability fails; the joint state is not wholly determined by the separate states.

Before we conclude that, after all, quantum mechanics is not a separable theory, we need to look a little deeper. We need to ask whether the apparent nonseparability of the quantum probabilities is compatible with an underlying separability. Can we, for example, regard the quantum statistics as averages over more basic states, which are themselves separable? This is exactly the sort of issue that the Bell theorem addresses, answering "no." But there the question is whether the conjunction of locality and factorizability can hold at the basic level. Here our concern is with maintaining the conjunction of locality and separability—especially in cases where the Bell inequalities, and hence factorizability, fail.

III. SEPARABILITY WITH LOCALITY: SOME PROOFS

Recall that we are considering correlation experiments violating the Bell inequalities that involve two measurements in each wing (2×2 experiments): call them A_1, A_2 in the A-wing and B_1, B_2 in the B-wing. In what follows, all models preserve locality. That means, for instance, that at each state λ the marginal distribution for A_1 in $A_1 \times B_1$ runs of the experiment is the same as the marginal distribution for A_1 in $A_1 \times B_2$ runs; similarly for the other three pairs. On this assumption, to build a model in which the composite statistics are separable is to define a probabilistic model involving basic states λ that reproduces the quantum statistics as averages over the λ. That model will identify for each state λ, the marginal distributions $p(A_i = x|\lambda)$, $p(B_j = y|\lambda)$ which we understand as the local states in each wing. Then we need to show that there is a single two-place function $F(.\ .)$ such that

$$F[p(A_i = x|\lambda), p(B_j = y|\lambda)] = p_\lambda{}^{AB}(x,y|i,j)$$

for all i, j, x, y, and λ. Thus a single F maps the pairs of marginals at λ to the joints at λ, so that, at every λ, the joint state is wholly determined by the local state. Finally we need to verify that the prescription for local states and joints does yield the quantum probabilities as averages over λ.

Below we take the basic states to be α and β and to begin with we assume these to be equiprobable. Let $a_{\alpha i} = p(A_i = 1|\ \alpha)$. So that $1 - a_{\alpha i} = p(A_i = -1|\alpha)$. Similarly let $b_{\alpha j} = p(B_j = 1|\alpha)$. If the statistics are separable, the A- and B-wing distributions at α are fixed by $a_{\alpha 1}$, $a_{\alpha 2}$ and $b_{\alpha 1}$, $b_{\alpha 2}$ respectively. In the same way we can let $a_{\beta i} = p(A_i = 1|\beta)$

and $b_{\beta j} = p(B_j = 1|\beta)$. Then the A- and B-wing distributions at β are fixed by $a_{\beta 1}$, $a_{\beta 2}$ and $b_{\beta 1}$, $b_{\beta 2}$.

If $a_{\alpha i} + a_{\beta i} = 1$, then

$$1/2[p(A_i = 1|\alpha) + p(A_i = 1|\beta)] = \langle p(A_i = 1|\lambda)\rangle = 1/2$$

which is the quantum probability that a measurement of A_i yields the result 1. Similarly, if $b_{\alpha i} + b_{\beta i} = 1$, then averaging over the two basic states yields the 50:50 quantum probabilities for each B-wing measurement to have 1 as a result.

Given quantum joint probabilities that violate the Bell inequalities, in order to build a separable model we will assign joint distributions at α and at β that have marginals as above (satisfying locality), and that average to the quantum joints. We will then show that there is a function F mapping the marginals at each of α and β to the joints assigned there, as required for separability. Here is an algorithm for how to do just that for almost all quantum statistics. That is, given "ensemble" joint probabilities, it is an algorithm for finding an α and β and an F that will do the job.

In a 2×2 experiment, quantum theory assigns joint distributions to the four pairs: A_i, B_j for $i,j = 1, 2$. The marginals are all probabilities of 0.5. So if we are given $P^{QM}(A_i{=}1, B_j{=}1)$, call it $P(A_iB_j)$, then we have an inversion symmetric distribution where

$$P^{QM}(A_i = -1, B_j = 1) = 1/2 - P(A_iB_j) = P^{QM}(A_i = 1, B_j = -1)$$

and

$$P(A_iB_j) = P^{QM}(A_i = -1, B_j = -1)$$

Thus $P(A_iB_j)$ is sufficient to determine the whole $P^{QM}(.\,.)$.

Then, where λ can be either α or β, assign at λ the joint distribution $p(.\,.\,|\lambda)$ defined as follows:

$$p(A_i{=}1, B_j = 1|\lambda) = P(A_iB_j) \text{ (that is, the quantum theory joint above)}$$

$$p(A_i{=}-1, B_j = 1|\lambda) = b_{\lambda j} - P(A_iB_j)$$

$$p(A_i{=}1, B_j = -1|\lambda) = a_{\lambda i} - P(A_iB_j)$$

$$p(A_i{=}-1, B_j = -1|\lambda) = 1 + P(A_iB_j) - a_{\lambda i} - b_{\lambda j}$$

One can check that these return the desired 50:50 marginals, respecting locality, and that averaging over $p(.\,.\,|\alpha)$ and $p(.\,.\,|\beta)$, so defined, produces exactly the quantum joints provided $a_{\beta i} = 1 - a_{\alpha i}$ and $b_{\beta j} = 1 - b_{\alpha j}$.

Given that each quantum joint satisfies $0 \leq P(A_iB_j) \leq 0.5$, and assuming that the marginals assigned at α and β take values between 0 and 1, the joint distribution $p(.\ .|\lambda)$ defined above requires

$$(1\lambda)\ P(A_iB_j) \leq a_{\lambda i}\ ,\ b_{\lambda j}$$

and

$$(2\lambda)\ a_{\lambda i} + b_{\lambda j} \leq 1 + P(A_iB_j)$$

Finally, then, let us choose the marginals. Here is how.

If among the numbers $P(A_iB_j)$ there is a 0 or 1/2, call it $P(A_1B_1)$ and rename the other three accordingly. We are going to assign marginal distributions in the A- and B-wings using the following scheme (in which it is useful to keep track of the four runs so that we can see that locality is satisfied):

For run $A_1 \times B_1$ at α : set $p(A_1{=}1|\alpha) = 0.5 = p(B_1{=}1|\alpha)$; that is, set $a_{\alpha 1} = 0.5 = b_{\alpha 1}$.
For run $A_1 \times B_2$ at α: set $a_{\alpha 1} = 0.5$ (locality!) and set $b_{\alpha 2} = x$, for some $0 \leq x \leq 1$ to be chosen below.
For run $A_2 \times B_1$ at α: set $a_{\alpha 2} = x$ and $b_{\alpha 1} = 0.5$ (locality).
For run $A_2 \times B_2$ at α: set $a_{\alpha 2} = x$ (locality) and $b_{\alpha 2} = x$ (locality).

The conditions (1α) and (2α) impose a set of inequalities that x must satisfy. Because of (1β) and (2β) these constraints are multiplied by the requirement that $a_{\beta i} = 1 - a_{\alpha i}$ and $b_{\beta j} = 1 - b_{\alpha j}$. We can summarize all of these inequalities as follows. Call $P(A_1B_2) = t$, $P(A_2B_1) = u$ and $P(A_2B_2) = v$. Then let

$$S = \max[\max(t,\ u\ ,v),\ 0.5 - \min(t,\ u,\ v/2)]$$

and let

$$T = \min[0.5 - \min(t,\ u,\ v/2),\ 1 - \max(t,\ u,\ v/2]$$

Since each joint $(t,\ u,\ v)$ is between zero and 0.5, it is easy to check that $S \leq T$. Then the number x must satisfy the condition that

$$S \leq x \leq T$$

To be specific we can choose $x{=}S$. Notice that if one (or more) of t, u, v is either 0 or 0.5 then $S = T = 0.5$ and in that case $x{=}0.5$. Excluding that case, we now conclude by defining the separating function F. Recall that F maps the marginals at λ to the joints at λ, for λ either α or β. So let F be defined by

$$F[a_{\lambda i}, b_{\lambda j}] = p(A_i{=}1,\ B_j{=}1|\lambda) = P(A_iB_j)$$

We need to check that this leads to no conflicts; that is, that F does not take the same input to different outputs. But the input pairs are just $<1/2,1/2>$, $<1/2,x>$, $<x, 1/2>$, $<x, x>$ for α and $<1/2,1/2>$, $<1/2,1-x>$, $<1-x, 1/2>$, $<1-x,1-x>$ for β. Provided $x \neq 1/2$ these are eight distinct pairs, so we can define F on them as indicated without fear of conflict. Obviously when we average over α and β we get back the original quantum joints. Thus, respecting locality, F maps the marginals in the A- and B-wings at λ, which define the states there, to joints at λ on the composite system. Averaged over α and β, these marginals, and the joints they determine via F, yield the single and joint probabilities that quantum theory assigns in the given experiment.

Consider a commonly used illustration for the violation of the Bell inequalities. It is the 2×2 experiment where $P(A_1B_1) = 0$ and $P(A_1B_2) = P(A_2B_1) = P(A_2B_2) = 3/8$. Here the angular separation is 0 between the A_1 setting and the B_1 setting, and all other separations are 120 degrees. Adding the three nonzero joints and subtracting the fourth produces 9/8, whereas the Bell inequality requires a result less than 1. An easy calculation yields that $S = 0.375$, which we choose as our "x" for the separating model. (Here $T = 0.625$.) So the Bell inequality fails but locality and separability both hold in our model. More significantly, consider the experiments at angular separations where the Bell inequality achieves its maximum violation, which is the type of experiment most frequently run and most thoroughly studied. We can take the angles to be 45 degrees between the A_1 setting and the B_1 setting, and 135 degrees for the other three. Then the quantum joint probabilities are $P(A_1B_1) = 0.073$ and $P(A_1B_2) = P(A_2B_1) = P(A_2B_2) = 0.427$. Adding the three big joints and subtracting the fourth little one produces 1.208, which exceeds the Bell limit of 1 by the maximum amount possible for experiments of this type. We can readily model the experiment in a local and separable way, however. S is simply 0.427, which we can take for our "x." (Here $T = 0.573$.)

Our demonstration of separability so far covers all 2×2 experiments where no more than one joint is either 0 or 1/2. That covers a lot of experiments but it does not include the tantalizing Hardy cases that also violate the Bell inequalities and which arise in virtually every interaction between spin-1/2 systems.[22] In the Hardy configuration we have $P(A_1B_1) = 0$, $P(A_1B_2) = 0.5 = P(A_2B_1)$ and $P(A_2B_2) = 0.09$. Our model breaks down here, for $S = T = x = 0.5$ which makes F many-one, and so not a function that determines the joints from the

[22] L. Hardy, "Nonlocality for Two Particles without Inequalities for Almost All Entangled States," *Physical Review Letters*, LXXI (1993): 1665–68.

marginals. Indeed, calculations show that no model like ours, where the distributions of the λ states is uniform, can separate the Hardy probabilities. But they can be separated nevertheless. Indeed, we will show how to separate any experiment where $P(A_1B_1) = 0$, $P(A_1B_2) = 0.5 = P(A_2B_1)$, and $P(A_2B_2) = H$ for any number H satisfying $0 < H < 0.5$. (For $H = 0$, the Bell inequalities hold, hence there is a local and factorizable model. At the other extreme, $H = 0.5$ describes an impossible configuration of angles; that is, no quantum experiment produces those numbers.)

Our Hardy model is similar to the one just described except that α now gets weight 0.25 and β gets weight 0.75. Moreover the 0 and 0.5 joint probabilities, working together, force a different configuration for the marginals. Again we separate the presentation into disjoint runs to show how locality is respected and also to make it easy to find the pairs of marginals on which F is defined.

For run $A_1 \times B_1$, at α set $a_{\alpha 1} = H$ and set $b_{\alpha 1} = 1-H$.
For run $A_1 \times B_2$, at α set $a_{\alpha 1} = H$ (locality) and set $b_{\alpha 2} = H$.
For run $A_2 \times B_1$, at α set $a_{\alpha 2} = 1-H$ and $b_{\alpha 1} = 1-H$ (locality).
For run $A_2 \times B_2$, at α set $a_{\alpha 2} = 1-H$ (locality) and $b_{\alpha 2} = H$ (locality).

Then at β we define $a_{\beta i}$ and $b_{\beta i}$ to satisfy

$$0.25a_{\alpha i} + 0.75a_{\beta i} = 0.5$$

$$0.25b_{\alpha i} + 0.75b_{\beta i} = 0.5$$

for $i = 1, 2$—so that the marginals averaged over α and β yield the quantum probabilities of 0.5. One can verify that for $0 < H < 0.5$, the marginal probabilities are between 0 and 1 and that the eight pairs of marginals (four at α and four at β) are all distinct. Finally, we assign the joints at α and β as follows:

$$F[a_{\lambda i}, b_{\lambda j}] = p(A_i{=}1, B_j{=}1|\lambda) = P(A_iB_j), \text{ for } \lambda = \alpha \text{ or } \beta \text{ and } i = j$$

$$F[a_{\lambda i}, b_{\lambda j}] = p(A_i{=}1, B_j{=}1|\lambda) = a_{\lambda i} \text{ for } \lambda = \alpha \text{ or } \beta \text{ and } i \neq j$$

For the case where $i \neq j$ the A and B marginals are the same for each λ, so if we pick one side (above we chose the A side) for the joints there and average over α and β, we get 0.5, since that is precisely how the α and β marginals were made to relate. One needs to check that all the inequalities required by (1λ) and (2λ) hold for these choices of marginals and joints, which they do. So, even the ubiquitous Hardy violations of the Bell inequalities can be modeled in such a way that the local states determine the joints.

IV. DISCUSSION

It is important to keep the quantifiers straight with respect to what we have shown. For each Bell-violating experiment considered above, there exists a function that can be used to determine the joint state from the separate states. This is sufficient to establish that each of those experiments can be given a local, separable model. Nevertheless, we have not proven the following: there exists a single function that can serve as a separations function for all those Bell-type experiments. Such a function would, so to speak, wholly replace the product function as a means of determining joint states from separate states in every single case.

It might be objected that a great deal hinges on this order of quantifiers. After all, one might argue, only if there were a single function could one use the separate states to predict what can be predicted on the basis of the whole state. Is that not, after all, what it means for the separate states *wholly to determine* the joint state?

We note, first of all, that what we have shown so far is already sufficient to refute the lemma according to which, in models of the sort considered above, factorizability was supposed to be a necessary condition for separability. Second, it is important to distinguish the issue of access or predictability from the metaphysical question of separability. Even if there were a universal separating function, we could still be stuck unless that function was computable, or otherwise suitable for making predictions. Looking back to Howard's discussion, we find that he says two inequivalent things about separability:

> The idea is that no information is contained in the joint state that is not already contained in the separate states, or *alternatively*, that no measurement result could be predicted on the basis of the joint state that could not be predicted on the basis of the separate states.[23]

The present context makes it clear, however, that it is one thing to say that the information is contained in the separate states, and quite another to say that one can extract the information and use it to make a prediction, or even that we could know in advance what the separating function will be.

Richard Healy[24] distinguishes these cases, defining what he calls *spatiotemporal separability* as follows:

[23] "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," p. 226n, our emphasis.

[24] "Holism and Nonseparability," this JOURNAL, LXXXVIII, 8 (August 1991): 393–421.

> The qualitative, intrinsic physical properties of a compound system are *supervenient* on the qualitative, intrinsic physical properties of its spatially separated component systems together with the spatial relations among these component systems (*ibid.*, p. 410).

The language of supervenience marks a criterion other than that of predictability. In fact, Healy contrasts the relevant principle with what he calls *explanatory principles of holism*—principles, which depend on predictability for their application.

Since the original inspiration for the concept of separability came from Einstein, perhaps it would be helpful to go back and look at what Einstein himself had in mind. He wrote:

> [Without a principle of separation] physical thought in the sense familiar to us would not be possible. Nor does one see *how physical laws could be formulated and tested* without such a clean separation.[25]

If Einstein's concern is for the testability of laws, then the concern would likely be along the following lines. We make measurements in spatially local contexts. If we are going to have a theory that we can test, then we had better be able to use the information that we gather locally to build up what we need to know in order to tell what a given theory predicts about the world. In other words, it had better be the case that everything that is predictively relevant about a system can be humanly put together from locally separate states of affairs. For if there are theoretically/predictively important aspects of a system that we cannot come to know by making local measurements, then we will not be able to know what outcome a theory predicts for a given state of affairs in important cases. Hence, we would not be able to test such a theory.

If the above reconstruction of Einstein's thoughts is roughly correct, then it should be clear that Einstein had in mind, after all, something over and above metaphysical separability. The concern that Einstein evinces for the testability of physical laws forces this conclusion. By itself, separability does not address Einstein's concern about how quantum mechanics could possibly work in practice. For that one needs a (metaphysical) separability that is also operational. That is, to satisfy Einstein, one needs more than a separating function for every experiment, or even a universal function. What one needs are humanly identifiable, computable functions.

But if Einstein advocated a metaphysical separability that was also

[25] Quoted in Howard, "Holism, Separability, and the Metaphysical Implications of the Bell Experiments," p. 233.

operational, because he believed that this was a necessary condition for the testability of a theory, and if he was right, then we are left with a strange conundrum. For quantum mechanics does not appear to be even metaphysically separable; forget about the operational criterion. So the conundrum is this: If, according to Einstein, separability is the *sine qua non* of a testable theory, then how does quantum mechanics work? That is to say, if quantum mechanics is a theory in which the principle of separation actually fails, how does it come not only to have been tested, but to have passed with flying colors? How does quantum mechanics do what Einstein denies can be done by any theory that violates the principle of separability?

Of course, we could conclude that Einstein was just wrong here. But assuming he was on the right track, a good way to approach this question is to re-examine the notion of "state" in the quantum theory. Thus far, we have taken a state to be a family of probability distributions. But there is an ambiguity here familiar to all students of mathematics: the ambiguity between the values of a function (what Gottlob Frege called "the course of values") and the function itself. In our models, and in discussing the quantum probabilities as states, we took the local states to be the bare distributions, which is to say the *values of the mapping*—from detector settings (or angles) to probability distributions on $\{1, -1\}$. But if we say that the local state is not the value of this mapping but the mapping itself, then we have a fresh way of regarding the quantum theory. Perhaps the best way to bring this out is to display the local states as a set of pairs $<z, P_z(.)>$, where z is an angle corresponding to one of the detector settings in a given wing and $P_z(.)$ is the marginal distribution on $\{1, -1\}$ at z. Then separability asks whether there is a function $F[.\ .]$ such that for all states $<a, P_a(.)>$ in the A-wing and $<b, P_b(.)>$ in the B-wing $F[<a, P_a(.)>, <b, P_b(.)>]$ is the joint state of the composite system. Put in this more perspicuous way, the answer is clear. There is such a universal separating function; namely,

$$F[<a, P_a(.)>, <b, P_b(.)>] = <<a, b>, 1/2\sin^2[(a-b)/2]>$$

the joint probability function in the singlet state that assigns to the angle a in the A-wing and b in the B-wing the probability $1/2\sin^2[(a-b)/2]$ for spin-up in both directions. This is, moreover, a simple function, easy to compute and easy to use for predictions. Notice that this F depends only on the domain of the marginal distributions (the angles a and b), not on the range. It must do so since all the distributions $P_a(.)$ and $P_b(.)$ are the same. Still, on the present, more careful conception of state, what the function F achieves is *wholly to determine* the state of the whole system from the state of the parts. Einstein's concerns are addressed in the best possible way. There is a universal separating function and one entirely suitable for predicting and testing. It would be hard to ask for more by way of quantum separability, and it would be hard to imagine a stronger defeat for the "lemma" that separability implies factorizability.

ERIC WINSBERG

University of South Florida/Tampa

ARTHUR FINE

University of Washington/Seattle